

Quantum criticality in inter-band superconductors

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In fermionic systems with different types of quasi-particles, attractive interactions can give rise to exotic superconducting states, as pair density wave (PDW) superconductivity and breached pairing. In the last years the search for these new types of ground states in cold atom and in metallic systems has been intense. In the case of metals the different quasi-particles may be the up and down spin bands in an external magnetic field or bands arising from distinct atomic orbitals that coexist at a common Fermi surface. These systems present a complex phase diagram as a function of the difference between the Fermi wave-vectors of the different bands. This can be controlled by external means, varying the density in the two-component cold atom system or, in a metal, by applying an external magnetic field or pressure. Here we study the zero temperature instability of the normal system as the Fermi wave-vectors mismatch of the quasi-particles (bands) is reduced and find a second order quantum phase transition to a PDW superconducting state. From the nature of the quantum critical fluctuations close to the superconducting quantum critical point (SQCP), we obtain its dynamic critical exponent. It turns out to be $z = 2$ and this allows to fully characterize the SQCP for dimensions $d \geq 2$.

In strongly correlated materials superconductivity can be suppressed in different ways. Most commonly, this is accomplished by an external magnetic field, applied pressure or doping¹⁻⁷. However, the point in the phase diagram where the critical temperature T_c vanishes as a function of the external parameters is not necessarily associated with a SQCP. In the case of superconductivity induced by antiferromagnetic fluctuations due to the proximity of an antiferromagnetic quantum critical point (AFQCP)⁸, as the system moves away from the AFQCP, these fluctuations change from attractive to repulsive and superconductivity just fades away⁹.

Inter-band superconductivity is a long standing problem in condensed matter physics^{10,11}. It can be realized, for example, by applying an external magnetic field in a metallic system. The field splits the band and the phase diagram as a function of the mismatch of the Fermi wave-vectors of the up and down spin bands can be investigated. It also occurs for the case of superconductivity mediated by valence fluctuations, where the most relevant correlations are due to inter-band pairing¹². In multi-band metals, pressure can be used to change the hybridization and the mismatch of the Fermi wave-vectors of the different bands coexisting at the common Fermi surface¹³. More recently this problem has received much attention due to the possibility of investigating it in cold atom systems. In this case the attractive interaction between two fermionic systems with different Fermi wave-vectors can be tuned by Feshbach resonance¹⁴. The Fermi wave-vector mismatch δk_F in this case is controlled by varying the density of the atoms and this allows the phase diagram to be fully explored. In the core of neutron stars, up and down quarks, in different numbers, with attractive interactions may give rise to color *inter-band* superconductivity^{13,14}. We neglect here orbital effects. Whenever a *magnetic field* is mentioned, it is to be considered as an external agent whose only effect

is to produce the mismatch of the Fermi wave-vectors. While avoiding complications related to the vortices¹⁵, the present approach is relevant for many interesting problems¹⁴ as pointed out above.

The zero temperature phase diagram for inter-band superconductors with s-wave pairing has been established using mean-field calculations¹¹. As the Fermi wave-vector mismatch increases, there is a first order phase transition from a BCS-like state to an FFLO or PDW state. As the mismatch further increases there is a continuous transition to a normal metallic phase¹¹. As new systems are discovered and the possibility of finding experimentally new exotic PDW phases increases, it is important to fully understand the nature of the normal to PDW transition beyond the mean-field approximation. Here we investigate this transition using a new approach. It is based on a perturbation theory for retarded and advanced Green's functions¹⁶. We relate the appearance of superconductivity in the multi-band system to the divergence of a generalized susceptibility¹⁷ $\chi(q, \omega)$, like in the Stoner criterion for ferromagnetism.

We start in the normal phase and calculate the response of the system to a frequency and wave-vector dependent *fictitious* external field which couples to the superconducting order parameter¹⁸. For simplicity we consider here the case of s-wave superconductivity.

In our approach, at the level of the RPA approximation presented here, we have to calculate only single-particle Green's functions. This is different and simpler than the usual linear response theory, which relates the response of the system to two-particles Green's functions¹⁹.

At zero temperature, as the Fermi wave-vectors mismatch is reduced, we find a divergence of the static part of the susceptibility at a finite wave-vector $q = q_0$. This occurs at the critical mean-field value of the mismatch which destroys pair density wave superconductivity. This divergence implies that even in zero *field* the system can

have a finite inhomogeneous superconducting order parameter. The condition for the superconducting instability can be expressed in the form of a Stoner-like criterion, $1 - U\chi_0(q = q_0, \delta k_F) = 0$. This determines either a critical value of the attractive interaction U_c above which the system is superconductor or a critical mismatch below which superconductivity sets in. The coupling of the order parameter to the electronic degrees of freedom determines the frequency dependence of the generalized susceptibility and the nature of the quantum fluctuations, i.e., the dynamic critical exponent z^{20} , at the PDW SQCP. The theory is equivalent to a quantum Gaussian approach and since the dynamic exponent turns out to be $z = 2$, it yields the correct description of the SQCP for dimensions $d \geq 2$.

We start with the following Hamiltonian describing a two-band system with inter-band attractive interactions,

$$\mathcal{H}_0 = \sum_{i,j} t_{ij}^a a_i^\dagger a_j + \sum_{i,j} t_{ij}^b b_i^\dagger b_j - U \sum_i n_i^a n_i^b. \quad (1)$$

For simplicity, we omitted spin indexes¹⁴. The bands a and b can be the up and down spin bands of a metal polarized by an external magnetic field h ¹¹, different types of atoms or the hybridized bands of a multi-band metal¹³. The interaction U is an attractive interaction between the fermions in different bands. We calculate the response of the normal two-band system to a wave-vector and frequency dependent fictitious field that couples to the superconducting order parameter of interest. In this case,

$$\mathcal{H}_1 = -g \sum_i e^{i\mathbf{q} \cdot \mathbf{r}_i} e^{i\omega_0 t} (a_i^\dagger b_i^\dagger + a_i b_i), \quad (2)$$

where the frequency ω_0 has a small positive imaginary part to guarantee the adiabatic switching on of the field. The response of the system to the fictitious field, will be obtained using perturbation theory for the retarded and advanced Green's functions¹⁶. We start in the normal phase where the superconducting order parameter is zero in the absence of the *external field* g . We split the Green's functions, normal and anomalous, in two contributions. The first of order zero and the second of first order in the field g . For the anomalous Green's functions $\ll a_{i\sigma}^\dagger | b_{j\sigma'}^\dagger \gg$, for example, we write,

$$\ll a_{i\sigma}^\dagger | b_{j\sigma'}^\dagger \gg \rightarrow \ll a_{i\sigma}^\dagger | b_{j\sigma'}^\dagger \gg^{(0)} + \ll a_{i\sigma}^\dagger | b_{j\sigma'}^\dagger \gg^{(1)}.$$

In the normal phase and in the absence of the fictitious field, the relevant zero order Green's functions can be easily calculated,

$$G_k^{aa}(\omega) = \ll a_k | a_{k'}^\dagger \gg_\omega^{(0)} = \frac{\delta_{k,k'}}{2\pi(\omega - \epsilon_k^a)},$$

$$G_k^{bb}(\omega) = \ll b_k | b_{k'}^\dagger \gg_\omega^{(0)} = \frac{\delta_{k,k'}}{2\pi(\omega - \epsilon_k^b)},$$

$$\ll a_k | b_{k'}^\dagger \gg_\omega^{(0)} = 0,$$

$$\ll a_k^\dagger | a_{k'}^\dagger \gg_\omega^{(0)} = 0,$$

$$\ll b_k^\dagger | b_{k'}^\dagger \gg_\omega^{(0)} = 0,$$

and

$$\ll a_k^\dagger | b_{k'}^\dagger \gg_\omega^{(0)} = \ll b_k^\dagger | a_{k'}^\dagger \gg_\omega^{(0)} = 0,$$

where $\epsilon_k^\alpha = \sum_j t_{ij}^\alpha e^{i\mathbf{k} \cdot (\mathbf{r}_j - \mathbf{r}_i)}$, ($\alpha = a, b$). For the first order Green's functions we get,

$$\begin{aligned} (\omega - \epsilon_k^a) \ll a_k | a_{k'}^\dagger \gg_\omega^{(1)} &= (U\delta\Delta_k^{ab} + g) \ll b_{k+q}^\dagger | a_{k'}^\dagger \gg_{\omega+\omega_0}^{(0)}, \\ (\omega - \epsilon_k^b) \ll b_k | b_{k'}^\dagger \gg_\omega^{(1)} &= (U\delta\Delta_k^{ab} - g) \ll a_{k+q}^\dagger | b_{k'}^\dagger \gg_{\omega+\omega_0}^{(0)}, \\ (\omega - \epsilon_k^a) \ll a_k | b_{k'}^\dagger \gg_\omega^{(1)} &= (U\delta\Delta_k^{ab} + g) \ll b_{k+q}^\dagger | b_{k'}^\dagger \gg_{\omega+\omega_0}^{(0)}, \\ (\omega + \epsilon_k^a) \ll a_k^\dagger | a_{k'}^\dagger \gg_\omega^{(1)} &= -(U\delta\Delta_k^{ab} + g) \ll b_{k+q} | a_{k'}^\dagger \gg_{\omega+\omega_0}^{(0)}, \\ (\omega + \epsilon_k^b) \ll b_k^\dagger | b_{k'}^\dagger \gg_\omega^{(1)} &= -(U\delta\Delta_k^{ab} - g) \ll a_{k+q} | b_{k'}^\dagger \gg_{\omega+\omega_0}^{(0)}, \\ (\omega + \epsilon_k^a) \ll a_k^\dagger | b_{k'}^\dagger \gg_\omega^{(1)} &= -(U\delta\Delta_k^{ab} + g) \ll b_{k+q} | b_{k'}^\dagger \gg_{\omega+\omega_0}^{(0)}. \end{aligned} \quad (3)$$

The first three first order propagators are written in terms of zero order propagators associated with superconductivity. Since we are analyzing the system in the normal state, these anomalous zero order propagators vanish and consequently the first order ones also vanish. The three last first order propagators are written in terms of conventional zero order propagators, but just the last one is different from zero. It is given by,

$$\ll a_k^\dagger | b_{k'}^\dagger \gg_\omega^{(1)} = -\delta_{k+q,k'} \frac{(U\delta\Delta_k^{ab} + g)}{2\pi(\omega + \epsilon_k^a)(\omega + \omega_0 - \epsilon_{k+q}^b)}. \quad (4)$$

We use the fluctuation-dissipation theorem¹⁶ to obtain the correlation function $\delta\Delta^{aa}$ from the above anomalous Green's functions, we get,

$$\delta\Delta^{ab} = \sum_k F_\omega \left\{ \ll a_k^\dagger | b_{k'}^\dagger \gg_\omega^{(1)} \right\},$$

where $F_\omega \{G(\omega)\} = -\int d\omega f(\omega) [G(\omega + i\epsilon) - G(\omega - i\epsilon)]$ is the statistical average of the discontinuity of the Green's functions $G(\omega)$ on the real axis¹⁶. The function $f(\omega)$ is the Fermi-Dirac distribution.

We define the generalized inter-band susceptibility by, $\chi_0^{ab}(q, \omega) = 2\pi \sum_k F_\omega \left\{ G_k^{aa}(-\omega) G_{k+q}^{bb}(\omega + \omega_0) \right\}$ or,

$$\chi_0^{ab}(q, \omega) = \frac{-1}{2\pi} \sum_k F_\omega \left[\frac{1}{(\omega + \epsilon_k^a)} \frac{1}{(\omega + \omega_0 - \epsilon_{k+q}^b)} \right]. \quad (5)$$

The superconducting response to the external field is obtained in the form,

$$\delta\Delta^{ab} = \frac{\chi_0^{ab}(q, \omega)}{1 - U\chi_0^{ab}(q, \omega)} g. \quad (6)$$

Finally, the susceptibility $\chi_0^{ab}(q, \omega)$, (Eq. 5), is given by,

$$\chi_0^{ab}(2q, \omega) = \frac{1}{2\pi} \sum_k \frac{1 - f(\epsilon_{k-q}^a) - f(\epsilon_{k+q}^b)}{\epsilon_{k-q}^a + \epsilon_{k+q}^b - \omega_0}. \quad (7)$$

Let us consider the case of a metal polarized by an external magnetic field h . In this case the a and b -bands are given by the dispersion relation of the up and down spin bands, respectively,

$$\epsilon_k^{a,b} = \frac{\hbar^2(k^2 - k_F^2)}{2m} \mp h,$$

where k_F is the original Fermi wave-vector of the unpolarized band ($h = 0$). The mismatch of the new Fermi wave-vectors is given by, $\delta k_F = k_F^a - k_F^b = h/v_F$, where v_F is the Fermi velocity. At zero temperature, the generalized susceptibility $\chi_0^{ab}(2q, \omega_0 = 0)$ can be calculated and we get,

$$\chi_0^{ab}(2q, 0) = \rho \left\{ 1 - \ln\left(\frac{h}{v_F k_C}\right) - \frac{1}{2} \left[\frac{1}{\bar{q}} \ln\left(\frac{\bar{q}+1}{\bar{q}-1}\right) + \ln(\bar{q}^2 - 1) \right] \right\}, \quad (8)$$

where $\bar{q} = v_F q/h = q/(k_F^a - k_F^b)$ and $\rho = 3/(8\pi^3 E_F)$ is the density of states of the unpolarized system. The quantum critical point separating the normal metal from the superconducting PDW phase is obtained from the condition, $1 - U\chi_0^{ab}(2q, 0) = 0$, which can be written as,

$$U\rho \left\{ 1 - \ln\left(\frac{h}{\Delta_0}\right) - \frac{1}{2} \left[\frac{1}{\bar{q}} \ln\left(\frac{\bar{q}+1}{\bar{q}-1}\right) + \ln(\bar{q}^2 - 1) \right] \right\} = 0, \quad (9)$$

where $\Delta_0 = v_F k_C \exp(-1/\rho U)$. Notice that a PDW state is only possible for $\bar{q} > 1$. The equation above determines the critical field h_N (or mismatch) for a fixed value of \bar{q} . This is given by,

$$\frac{h_N}{\Delta_0} = \frac{e}{(1 + \bar{q})} \left(\frac{\bar{q} + 1}{\bar{q} - 1} \right)^{\frac{\bar{q}-1}{2\bar{q}}}. \quad (10)$$

The maximum value of h_N for which the instability occurs is denoted by h_C . It is easily obtained by differentiating $h_N(\bar{q})$ with respect to \bar{q} . From the equation $(\partial \ln h_N / \partial \bar{q})_{\bar{q}=\bar{q}_C} = 0$, we find that:

$$\bar{q}_C = \frac{1}{2} \ln \left(\frac{\bar{q}_C + 1}{\bar{q}_C - 1} \right).$$

This gives $\bar{q}_C \cong 1.2$, that substituted in Eq. 10 for h_N yields $h_C = h_N(\bar{q}_C) \cong 1.5\Delta_0$. This agrees with the result of Takada and Izuyama²² for the vanishing of the FFLO phase starting from this phase.

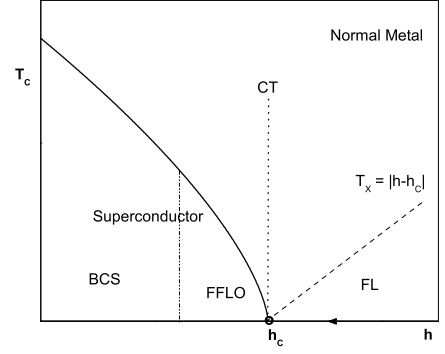


FIG. 1: Schematic $T_c \times h$ phase diagram for the inter-band superconductor. At zero temperature, as the external magnetic field or Fermi wave-vectors mismatch is reduced (indicated by the arrow), the normal metal becomes unstable to an inhomogeneous superconducting phase. This is signaled by the divergence of the q -dependent generalized susceptibility. The FFLO superconductor quantum critical point at h_C has a dynamic exponent $z = 2$ and this allows to determine the behavior of the thermodynamic quantities along the critical trajectory (CT) as given in the text. As the external field is further reduced there is a first order transition from the FFLO state to a BCS superconductor¹³ which is not studied here. The crossover line T_\times marks the onset of Fermi liquid behavior with decreasing temperature.

As discussed in Ref.¹³ an FFLO phase can also be induced by hybridization (V). The results above can be easily extended for the case of hybrid bands¹³. The Fermi wave-vectors mismatch is given by, $k_F^a - k_F^b = 4V/v_F(1 + \alpha)$, where $\alpha < 1$ is the ratio of the effective masses of the quasi-particles. The mismatch is proportional to the hybridization V (or pressure), which now plays the role of the magnetic field.

In order to study the fluctuations close to the PDW SQCP we have to expand Eq. 7 for small frequencies ω_0 , $h \cong h_C$ and $q \cong q_C$. At the same level of approximation for q of Eq. 8, we get for the denominator of the generalized susceptibility in Eq. 6,

$$1 - U\chi_0^{ab}(2q, \omega_0) = U\rho \left[\frac{h - h_C}{h_C} + i \frac{\omega_0}{v_F q_C} + \frac{1}{q_C^2 - 1} (q - q_C)^2 \right]. \quad (11)$$

The coupling of the superconductor order parameter to the electronic degrees of freedom gives rise to Landau damping of the superconductor quantum fluctuations. These modes are purely evanescent with an imaginary dispersion relation.

As in Hertz approach²⁰ to quantum phase transitions, we can construct from the dynamical susceptibility a quantum Gaussian action for this problem²¹, which can be written as,

$$\mathcal{S} = \int d^d q \int d\omega [g + \omega + q^2] |\psi(q, \omega)|^2, \quad (12)$$

where $g = h - h_C$ measures the distance to the PDW SQCP, $\psi(q, \omega)$ is the PDW superconductor order param-

eter and $q - q_C \rightarrow q$. The scaling properties of the free energy associated with this action allows to identify the dynamic exponent of the SQCP at which the FFLO instability occurs. This is easily found as $z = 2$. The knowledge of the dynamic exponent and the quantum hyperscaling relation^{21,23}, $2 - \alpha = \nu(d + z)$, allows to fully characterize the universality class of the FFLO quantum phase transition. Since $d_{eff} = d + z$, for $d = 2$ and $d = 3$, the critical exponents are Gaussian with possible logarithmic corrections for $d = 2$. The reason is that for $d \geq 2$, interaction between the fluctuations are irrelevant in the renormalization group sense and the Gaussian action, Eq. 12, gives the correct description of the quantum phase transition²¹.

Next step in the calculations is to extend the results to finite temperatures and particularly to obtain the correction to the mean-field value of the shift exponent ψ of the critical line at very low temperatures close to the PDW SQCP. This can be carried out in different ways²⁴, such that, $T_C(h) \propto |h - h_C|^{1/\psi}$. It turns out that²⁴ $\psi = z/(d+z-2)$ for $d_{eff} > 4$ and can be determined from our knowledge of the dynamic exponent, which yields $\psi = 2/d$. In the same way, scaling²¹ gives the contribution of quantum critical fluctuations to the specific heat, $C/T \propto T^{(d-2)/2}$, along the critical trajectory (CT) shown in Fig. 1. Also shown in this figure is the crossover line²¹, $T_\times \propto |h - h_C|$, below which the system enters a Fermi liquid regime. We have used that $\nu z = 1$ for $d \geq 2$.

We have studied the zero temperature instability of a normal two-band system with inter-band attractive inter-

actions as the Fermi wave-vectors mismatch is reduced. The model describes a superconductor in an external field in the absence of orbital effects, a cold atom system with two atomic species or a two-band metal with mixing tuned by pressure. We find that as the mismatch of the Fermi wave-vectors is reduced, the system becomes unstable to an inhomogeneous, FFLO-like, superconducting phase characterized by a wave-vector $\bar{q} = \bar{q}_C$. We have introduced a new method to deal with systems coupled to a space and time dependent *external field* and respond adiabatically to this perturbation. The appearance of superconductivity is related to the divergence of a static generalized susceptibility for $\bar{q} = \bar{q}_C$ at a SQCP. The results obtained are valid to first order in the perturbation and coincide with those of linear response. However, in our approach the basic elements to be calculated are single particle Green's functions and not the usual two particle propagators of linear response theory. The present theory extends the mean-field treatment to include fluctuations close to this SQCP. We have obtained the dynamical critical exponent at the PDW SQCP and from that we have identified the universality class of this quantum critical point for $d \geq 2$. This allows several predictions to be made for the thermodynamic and transport properties close to the quantum phase transition and for the shape of the critical line for dimensions $d \geq 2$.

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